
***New Educational Functioning Level Descriptors for
Adult Basic Education (ABE)¹***

Mathematics

Introduction and Process

The Educational Functioning Level (EFL) Descriptors for Mathematics are written using the College and Career Readiness Standards for Adult Education (CCR) as the foundation. They are intended to guide both teaching and assessment for adult learners. While these narrative descriptors address the most critical concepts for adult learners (as defined in the Major Work of the Level), there are additional concepts found in the CCR standards that support the major work for each level, and that are included in these descriptors. Lesson plans and assessment items for adult learners should be based on the full text of the CCR standards for each level, using these critical concepts as the foundation for lesson development and assessment.

The mathematics descriptors are divided into six educational functioning levels. The levels are Beginning Literacy (corresponding to Level A of the CCR); Beginning Basic (corresponding to Level B of the CCR); Low Intermediate (corresponding to Level C of the CCR); Middle Intermediate (corresponding to part of the Level D CCR), High Intermediate (corresponding to the remainder of the Level D CCR); and Adult Secondary (corresponding to Level E of the CCR). Each of the levels corresponds roughly to two grade levels, in K-12 terms, except for Level E, which combines the critical concepts of all of grades 9 through 12. Within each level the descriptors are further divided by domain: *The Mathematical Practices, Number Sense and Operations, Algebraic Thinking, Geometry (and Measurement), and Data Analysis (Statistics and Probability)*.

The descriptors do not provide a complete or comprehensive delineation of all of the skills at any given level but provide examples of the most critical concepts and skills for the level to guide assessment and instruction. Assessment of the Mathematical Practice descriptors are best performed in the classroom using assessments that could be formative or summative and may be informal. It should be noted that mathematics placement decisions should take into account the reading level of the adult student. Verbally presented application problems at all mathematics levels require a minimum reading level.

Level 1: Beginning Literacy

The Mathematical Practices: Students prepared to exit this level are able to decipher a simple problem presented in a context and reason about and apply correct units to the results. They can visualize a situation using manipulatives or drawings and explain their processes and results using mathematical terms and symbols appropriate for the level. They recognize errors in the work and reasoning of others. They are able to strategically select and use appropriate tools to aid in their work, such as pencil/paper, measuring devices, and/or manipulatives. They can see

¹ These descriptors will not be implemented until the Secretary has determined that there is at least one assessment that is both aligned with the revised descriptors and that is suitable for use in the NRS. Until that time, programs will continue to use the existing NRS educational functioning level descriptors.

patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

Number Sense and Operations: Students prepared to exit this level have an understanding of whole number place value for tens and ones and are able to use their understanding of place value to compare two-digit numbers. They are able to add whole numbers within 100 and explain their reasoning, e.g., using concrete models or drawings and strategies based on place value and/or properties of operations. They are able to apply their knowledge of whole number addition and subtraction to represent and solve word problems that call for addition of three whole numbers whose sum is less than 20 by using such problem-solving tools as objects, drawings, and/or simple equations.

Algebraic Thinking: Students prepared to exit this level understand and apply the properties of operations to addition and subtraction problems. They understand the relationship between the two operations and can determine the unknown number in addition or subtraction equations.

Geometry and Measurement: Students prepared to exit this level can analyze and compare 2-dimensional and 3-dimensional shapes based on their attributes, such as their shape, size, orientation, the number of sides and/or vertices (angles), or the lengths of their sides. They can reason with two-dimensional shapes (e.g., quadrilaterals and half- and quarter-circles) and with three-dimensional shapes (e.g., right prisms, cones, and cylinders) to create composite shapes. They are able to measure the length of an object as a whole number of units, which are not necessarily standard units, for example measuring the length of a pencil using a paper clip as the length unit.

Data Analysis: Students prepared to exit this level are able to organize, represent, and interpret simple data sets (e.g., lists of numbers, shapes, or items) using up to three categories. They can answer basic questions related to the total number of data points in a set and the number of data points in each category, and can compare the number of data points in the different categories.

Level 2: Beginning Basic

The Mathematical Practices: Students prepared to exit this level are able to decipher two-step problems presented in a context, visualizing a situation using diagrams or sketches, and reasoning about and applying the correct units and the proper degree of precision to the results. They can explain their processes and results using mathematical terms and symbols appropriate for the level and recognize errors in the reasoning of others. They strategically select and use the appropriate tools to aid in their work, such as pencil/paper, measuring devices, manipulatives, and/or calculators. They are able to see patterns and structure in sets of numbers, including in multiplication or addition tables, and use those insights to work more efficiently.

Number Sense and Operations: Students prepared to exit this level understand place value for whole numbers to 1000 and can use that understanding to read, write, count, compare, and round three-digit whole numbers to the nearest 10 or 100. They are able to compute fluently with all four operations with whole numbers within 100. They use place value and properties of operations to explain why addition and subtraction strategies work, and can demonstrate an understanding of the inverse relationship between multiplication and division. They can solve

one- and two-step word problems involving all four operations within 100 and identify and explain arithmetic patterns. They have an understanding of fractions, especially unit fractions, and can represent simple fractions on a number line. They understand and can explain equivalence of fractions, can recognize and generate simple equivalent fractions, and can compare two fractions with the same numerator or denominator by reasoning about their size.

Algebraic Thinking: Students prepared to exit this level apply the properties of operations to multiplication and division of whole numbers. They understand the relationship between multiplication and division and can determine the unknown number in multiplication or division equations.

Geometry and Measurement: Students prepared to exit this level are able to reason about geometric shapes and their attributes. They can demonstrate an understanding that different shapes might share common attributes (e.g., four sides) and can compare and classify two-dimensional shapes, particularly quadrilaterals. They are able to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole. They can use common U.S. Customary and metric units for linear measurements (e.g., inches, feet, centimeters, and meters) and solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. They understand the concept of area and can relate it to addition and multiplication to solve real-world problems. They also understand, and can solve, real-world and mathematical problems involving perimeter of polygons.

Data Analysis: Students prepared to exit this level are able to draw and interpret simple graphs (e.g., bar graphs, picture graphs, and number line diagrams) including scaled bar and picture graphs. They can solve one- and two-step problems using scaled bar graphs. They can generate measurement data by measuring lengths to the nearest half- and quarter-inch and display that data by making a line plot marked off in appropriate units.

Level 3: Low Intermediate

The Mathematical Practices: Students prepared to exit this level are able to decipher multi-step problems presented in a context and reason about and apply the correct units and the proper degree of precision to the results. They can visualize a situation using diagrams or sketches, see multiple strategies for solving a problem, explain their processes and results, and recognize errors in the work and reasoning of others. They can express themselves using mathematical terms and notation appropriate for the level and can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, and/or technology. They are able to see patterns and structure in sets of numbers and geometric shapes and use those insights to work more efficiently.

Number Sense and Operations: Students prepared to exit this level understand place value for both multi-digit whole numbers and decimals to thousandths, and use their understanding to read, write, compare, and round decimals. They are able to use their place value understanding and properties of operations to fluently perform operations with multi-digit whole numbers and decimals. They can find common factors, common multiples, and understand fraction concepts, including fraction equivalence and comparison. They can add, subtract, multiply and divide with fractions and mixed numbers. They are able to solve multi-step word problems posed with

whole numbers and fractions, using the four operations. They also have an understanding of ratio concepts and can use ratio language to describe a relationship between two quantities, including the concept of a unit rate associated with a ratio.

Algebraic Thinking: Students prepared to exit this level are able to apply and extend their understanding of arithmetic to algebraic expressions, using a symbol to represent an unknown value. They can write, evaluate, and interpret expressions and equations, including expressions that arise from formulas used in real-world problems. They can solve real-world and mathematical problems by writing and solving simple one-variable equations and write a simple inequality that represents a constraint or condition in a real-world or mathematical problem. They can represent and analyze quantitative relationships between dependent and independent variables.

Geometry and Measurement: Students prepared to exit this level have a basic understanding of the coordinate plane and can plot points (i.e., ordered pairs) and place polygons in the coordinate plane to solve real-world and mathematical problems. They can classify two-dimensional shapes and use formulas to determine the area of two-dimensional shapes such as triangles and quadrilaterals. They can determine the surface area of three-dimensional shapes composed of rectangles and triangles, and find the volume of right rectangular prisms. They are able to convert like measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions to solve multi-step, real-world problems. They are also able to solve measurement word problems (such as those that involve area, perimeter, distance, time intervals, liquid volumes, mass, and money) that involve simple fractions or decimals.

Data Analysis and Statistics: Students prepared to exit this level have a basic conceptual understanding of statistical variability, including such concepts as center, spread, and the overall shape of a distribution of data. They can present data using displays such as dot plots, histograms, and box plots.

Level 4: Middle Intermediate

The Mathematical Practices: Students prepared to exit this level are able to think critically, determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and persevere in solving challenging problems. They can express themselves using the mathematical terms and notation appropriate to the level. They are able to defend their findings and critique the reasoning of others. They are accurate in their calculations and use estimation strategies to assess the reasonableness of their results. They can create algebraic and geometric models and use them to answer questions and solve problems. They can strategically select and use tools to aid in their work, such as pencil/paper, measuring devices, calculators, and/or spreadsheets. They are able to see patterns and structure in number sets, data, expressions and equations, and geometric figures.

Number Sense and Operations: Students prepared to exit this level have an understanding of the rational number system, including how rational numbers can be represented on a number line and pairs of rational numbers can be represented on a coordinate plane. They can apply the concept of absolute value to find horizontal and vertical distances. They are able to apply the properties of integer exponents and evaluate, estimate, and compare simple square roots and

cube roots. Individuals at this level also understand ratio, rate, and percent concepts, as well as proportional relationships.

Algebraic Thinking: Students prepared to exit this level understand the connections between proportional relationships, lines, and linear equations. They understand numerical and algebraic expressions, and equations and are able to use them to solve real-world and mathematical problems. They are able to analyze and solve linear equations and pairs of simultaneous linear equations. Individuals at this level are able to define, interpret, and compare linear functions.

Geometry: Students prepared to exit this level can solve real-world and mathematical problems that involve angle measure, circumference, and area of 2-dimensional figures. They are able to solve problems involving scale drawings of 2-dimensional geometric figures. They understand the concepts of congruence and similarity with respect to 2-dimensional figures. They understand the Pythagorean theorem and can apply it to determine missing lengths in right triangles.

Statistics and Probability: Students prepared to exit this level can summarize and describe numerical data sets in relation to their context, including determining measures of center and variability and describing patterns and/or striking deviations from patterns. They understand and can apply the concept of chance, or probability. They are able to use scatter plots for bivariate measurement data to describe patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association).

Level 5: High Intermediate

The Mathematical Practices: Students prepared to exit this level are able to think critically, determine an efficient strategy (from among multiple possible strategies) for solving a multi-step problem, and persevere in solving challenging problems. They can reason quantitatively, including using units as a way to solve problems. They are able to defend their findings and critique the reasoning of others. They are accurate in their calculations and use estimation strategies to assess the reasonableness of their results. They can create algebraic and geometric models and use them to answer questions and solve problems. They can strategically select and use tools to aid in their work, such as graphing calculators, spreadsheets, and/or computer software. They are able to make generalizations based on patterns and structure they discover in number sets, data, expressions and equations, and geometric figures and use these insights to work more efficiently.

Number Sense and Operations: Students prepared to exit this level can reason about and solve real-world and mathematical problems that involve the four operations with rational numbers. They can apply the concept of absolute value to demonstrate on a number line their understanding of addition and subtraction with negative and positive rational numbers. Individuals at this level can apply ratio and percent concepts, including using rates and proportional relationships to solve multi-step real-world and mathematical problems.

Algebraic Thinking: Students prepared to exit this level are able to use algebraic and graphical representations to solve real-world and mathematical problems, involving linear equations,

inequalities, and pairs of simultaneous linear equations. Individuals at this level are able to use linear functions to describe, analyze, and model linear relationships between quantities.

Geometry: Students prepared to exit this level can solve real-world and mathematical problems that involve volume and surface area of 3-dimensional geometric figures. They can use informal arguments to establish facts about various angle relationships such as the relationships between angles created when parallel lines are cut by a transversal. They apply the Pythagorean theorem to determine lengths in real-world contexts and distances in the coordinate plane.

Statistics and Probability: Students prepared to exit this level can use random sampling to draw inferences about a population and are able to draw informal comparative inferences about two populations using measures of center and measures of variability for numerical data from random samples. They can develop, use, and evaluate probability models. They are able to use scatter plots for bivariate measurement data to interpret patterns of association between two quantities (such as clustering, outliers, positive or negative association, linear or non-linear association) and a 2-way table to summarize and interpret bivariate categorical data.

Level 6: Adult Secondary

The Mathematical Practices: Students prepared to exit this level are able to think critically, make assumptions based on a situation, select an efficient strategy from multiple possible problem-solving strategies, plan a solution pathway, and make adjustments as needed when solving problems. They persevere in solving challenging problems, including considering analogous, simpler problems as a way to solving a more complex one. They can reason quantitatively, including through the use of units, and can express themselves using the precise definitions and mathematical terms and notation appropriate to the level. They are accurate in their calculations, use an appropriate level of precision in finding solutions and reporting results, and use estimation strategies to assess the reasonableness of their results. They are able to make conjectures, use logic to defend their conclusions, and can detect faulty thinking and errors caused by improper use of technology. They can create algebraic and geometric models and use them to answer questions, interpret data, make predictions, and solve problems. They can create algebraic and geometric models and use them to answer questions, interpret data, make predictions, and solve problems. They can strategically select and use tools, such as measuring devices, calculators, spreadsheets, and/or computer software, to aid in their work. They are able to see patterns and structure in calculations, expressions, and equations and make connections to algebraic generalizations, which they use to work more efficiently.

Number Sense and Operations: Students prepared to exit this level have extended their number sense to include irrational numbers, radicals, and rational exponents and understand and use the set of real numbers. They are able to assess the reasonableness of calculation results based on the limitations of technology or given units and quantities and give results with the appropriate degree of precision.

Algebraic Thinking: Students prepared to exit this level understand the structure of expressions and can use that structure to rewrite linear, exponential, and quadratic expressions. They can add, subtract, and multiply polynomials that involve linear and/or quadratic expressions. They are also able to create linear equations and inequalities and quadratic and simple exponential

equations to represent relationships between quantities and can represent constraints by linear equations or inequalities, or by systems of linear equations and/or inequalities. They can interpret the structure of polynomial and rational expressions and use that structure to identify ways to rewrite and operate accurately with them. They can add, subtract, and multiply polynomials that extend beyond quadratics. They are able to rearrange formulas to highlight a quantity of interest, for example rearranging Ohm's law, $V = IR$, to highlight resistance R . They are also able to create equations and inequalities representing relationships between quantities, including those that extend beyond equations or inequalities arising from linear, quadratic, and simple exponential functions to include those arising from simple rational functions. They are able to use these equations/inequalities to solve problems both algebraically and graphically. They can solve linear equations and inequalities; systems of linear equations; quadratic, simple rational, and radical equations in one variable; and recognize how and when extraneous solutions may arise.

Students prepared to exit this level also have a basic understanding of functions, can use function notation properly, and use such notation to write a function describing a relationship between two quantities. They are able to evaluate functions for inputs in their domains and interpret linear, quadratic, and exponential functions that arise in applications in terms of the context. They are able to construct, graph, compare, and interpret functions (including, but not limited to, linear, quadratic, and exponential). They can sketch graphs given a verbal description of the relationship and identify and interpret key features of the graphs of functions that arise in applications in a context. They are able to select or define a function that appropriately models a relationship and to compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

Geometry: Students prepared to exit this level can solve problems involving similarity and congruence criteria for triangles and use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. They can apply the concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTU's per cubic foot).

Data Analysis and Statistics: Students prepared to exit this level can summarize, represent, and interpret data based on two categorical and quantitative variables, including by using frequency tables. They can compare data sets by looking at commonalities and differences in shape, center, and spread. They can recognize possible associations and trends in data, in particular in linear models, and distinguish between correlation and causation. They interpret one- and two-variable data, including those with linear and non-linear relationships. They interpret the slope (rate of change) and intercept (constant term) for a line of best fit and in the context of the data. They understand and account for extreme points of data in their analysis and interpret relative frequencies (joint, marginal and conditional).